Synthetic-Inductive Category Theory

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This talk corresponds to [Neu25, Chapter 4], draft available at

jacobneu.com/PhD

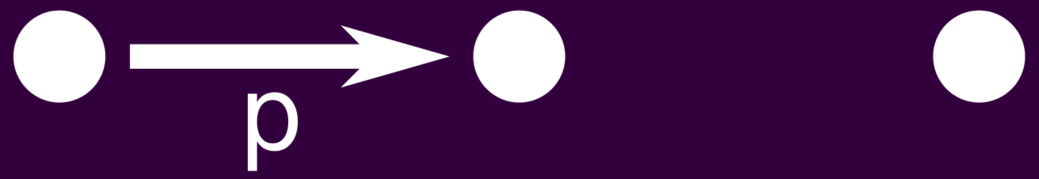
Martin Löf Type Theory: Synthetic groupoid theory

Synthetic groupoid structure of identity types

$$\frac{}{t : A}$$

Synthetic groupoid structure of identity types

$$\frac{t : A}{\mathsf{refl}_t : \mathsf{Id}(t, t)}$$



$$p : \mathrm{Id}(t, t')$$

Synthetic groupoid structure of identity types

$$\bullet \xrightarrow[p]{} \bullet \xrightarrow[q]{} \bullet$$

$$\begin{array}{c} p\colon \mathrm{Id}(t, t') \\ q\colon \mathrm{Id}(t', t'') \\ \hline \end{array}$$

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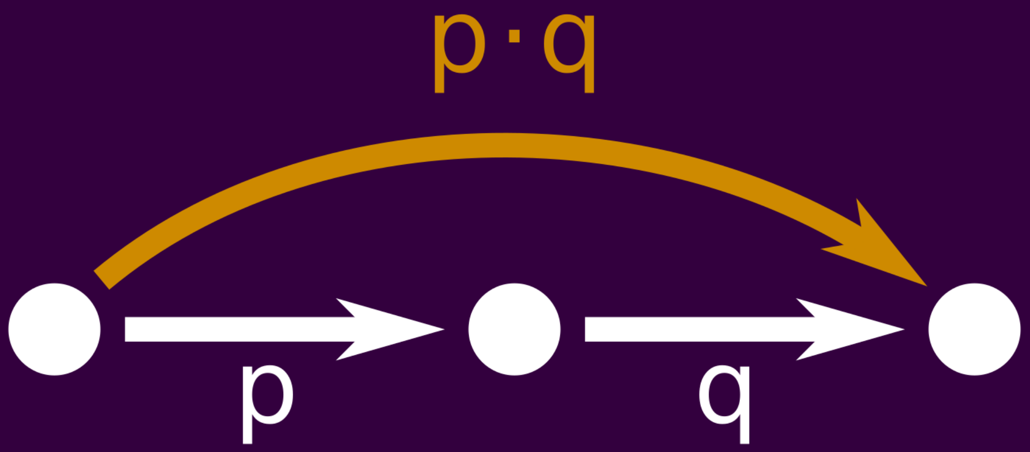

$$\frac{p : \mathsf{Id}(t, t') \qquad q : \mathsf{Id}(t', t'')}{p \cdot q : \mathsf{Id}(t, t'')}$$

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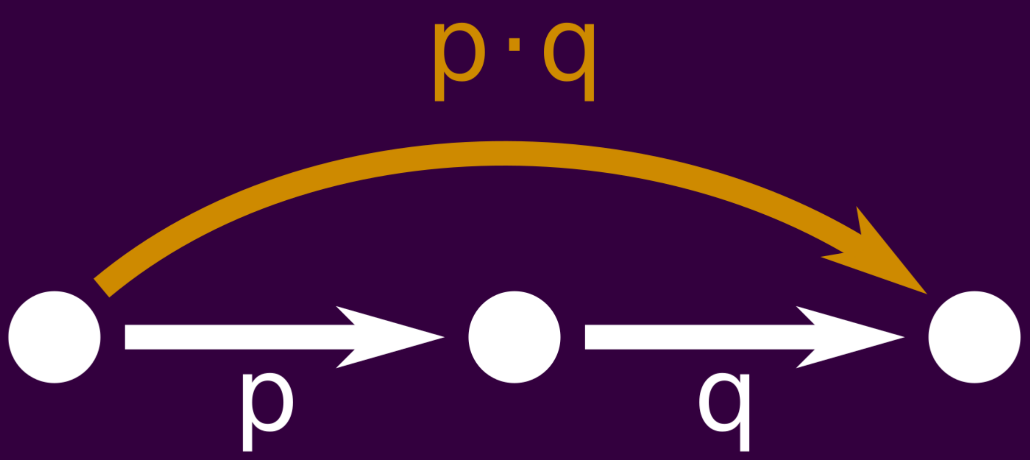

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$$p \cdot q := \mathsf{J}_{\mathsf{Id}(t,\_)}\, p\, (t'', q)$$

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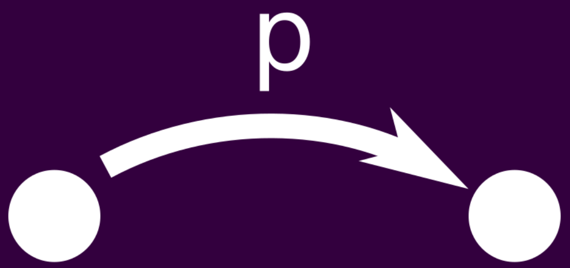

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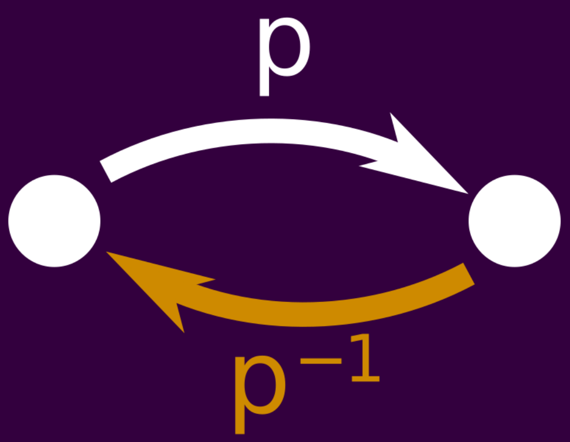

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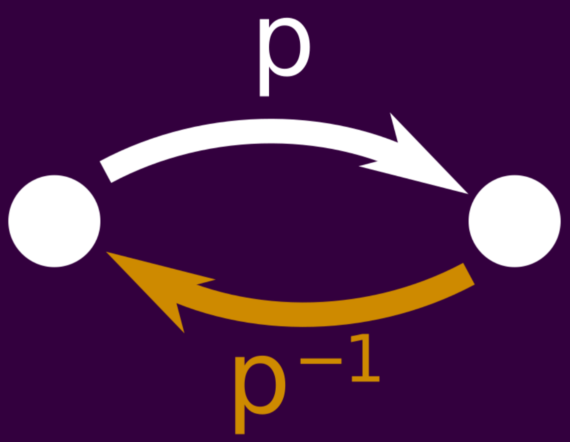

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Directed TT:

Synthetic *category* theory

Synthetic category structure of hom-types

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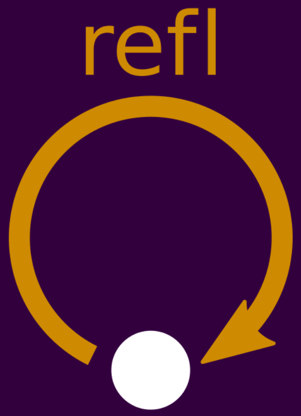
Synthetic category structure of hom-types



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Synthetic category structure of hom-types

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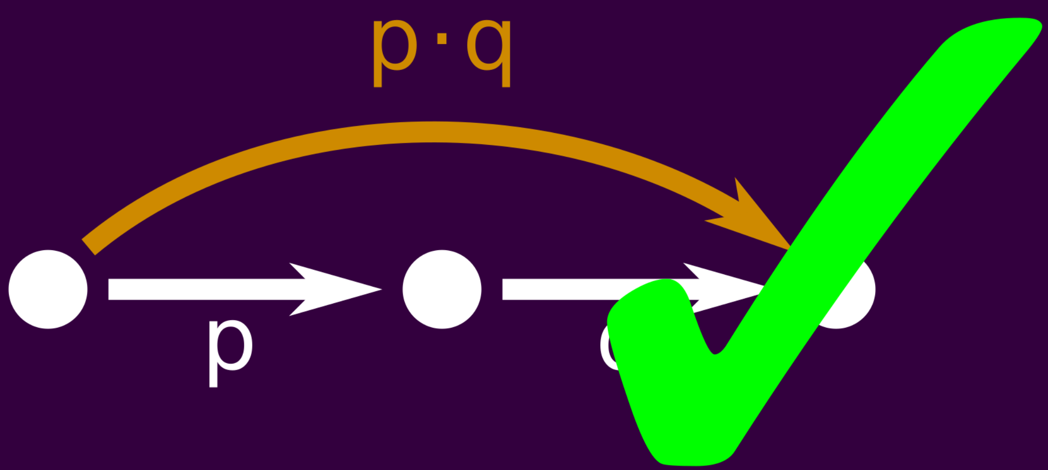

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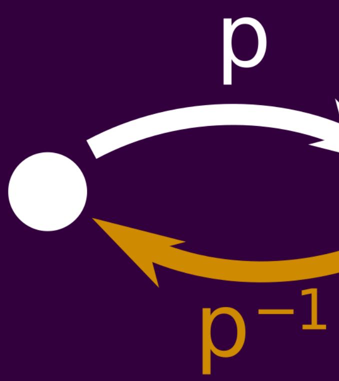

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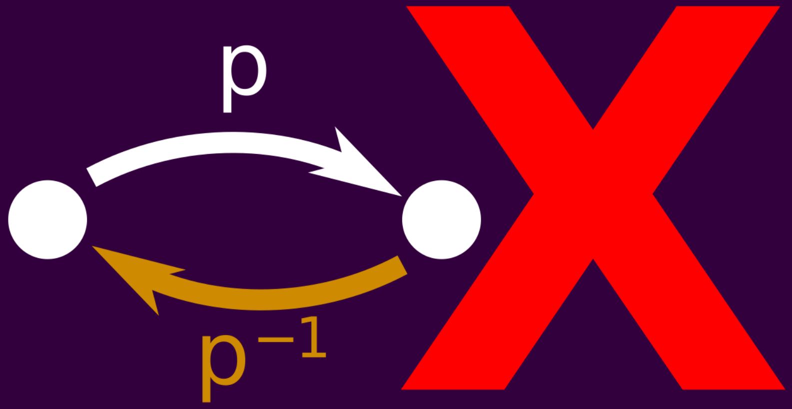

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Problem of Directed TT: **Make symmetry unprovable**

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Synthetic categories that aren't (necessarily) synthetic groupoids

Polarized and Directed type theory

We have **polarity annotations** on our types to mark co- or contra-variance

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The polarity annotations allow us to properly state the variance of hom-sets [Nor19]:

$$\frac{\Delta \vdash t : A^{-} \qquad \Delta \vdash t' : A}{\Delta \vdash \mathrm{Hom}(t, t') \ \mathbf{type}}$$

Closed term coercion

For **closed** terms, we can coerce between A and A^- [NA25, Neu25]:

$$\frac{t : A^-}{-t : A} \qquad \frac{}{- - t = t}$$

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A category and its opposite have the same objects

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Key Point Cannot (in general) negate open terms

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The coercions make it possible to introduce refl:

$$\frac{t : A^-}{\mathrm{refl}_t : \mathrm{Hom}(t, -t)}$$

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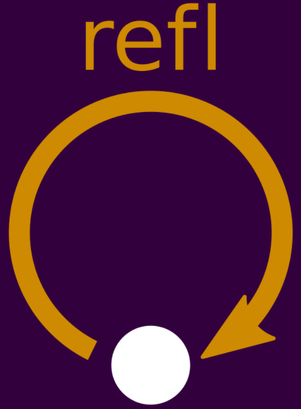

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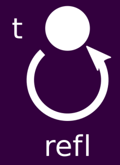
Coslice Path Induction

$$t : A^{-}$$



Coslice Path Induction

$$t : A^{-}$$
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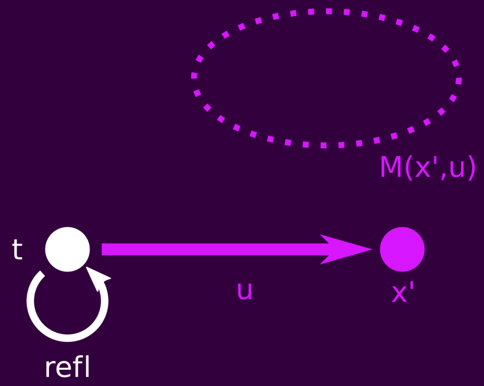

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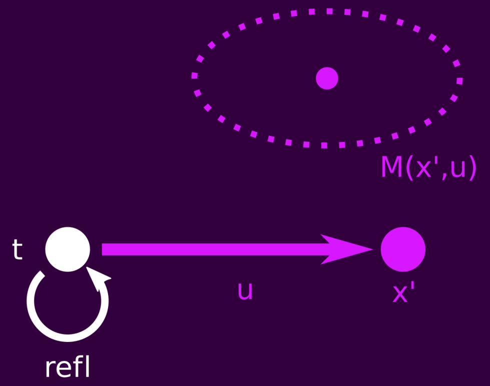

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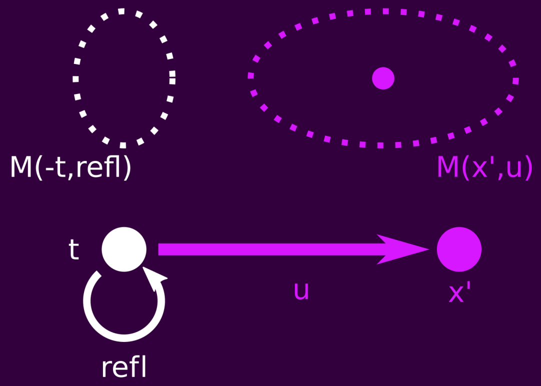

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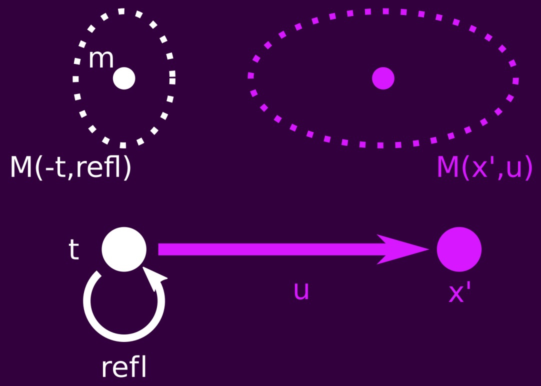

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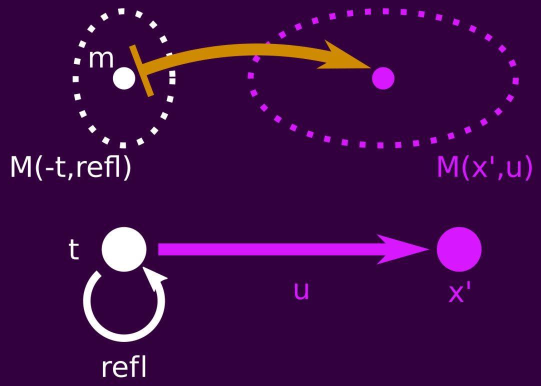

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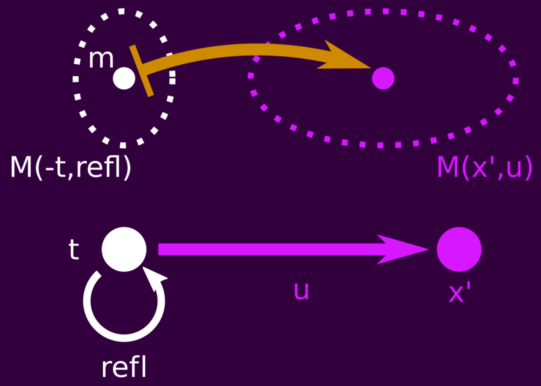

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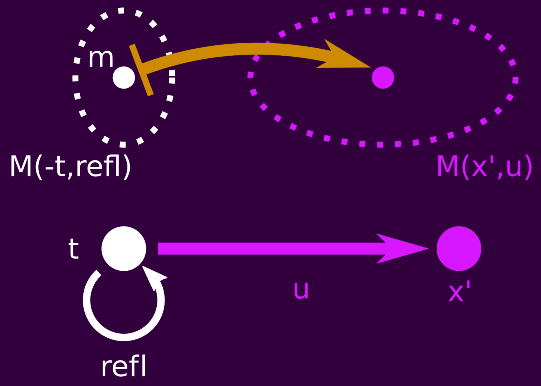

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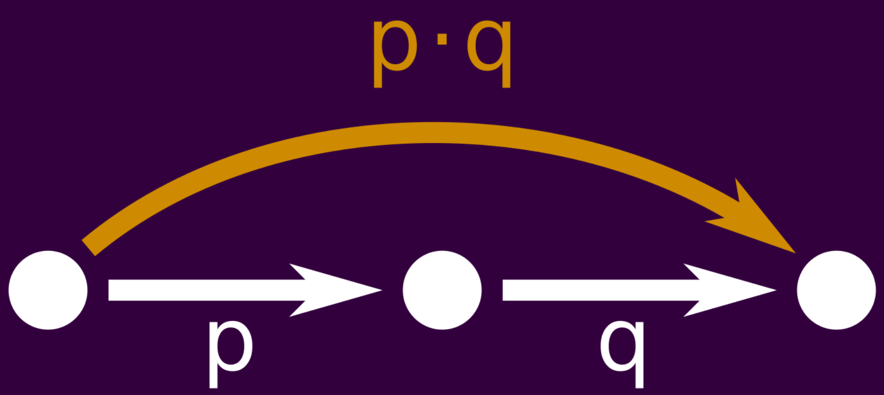
refl_t is the “universal coslice” under t

This solves the fundamental problem of directed TT



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Synthetic category structure of hom-types

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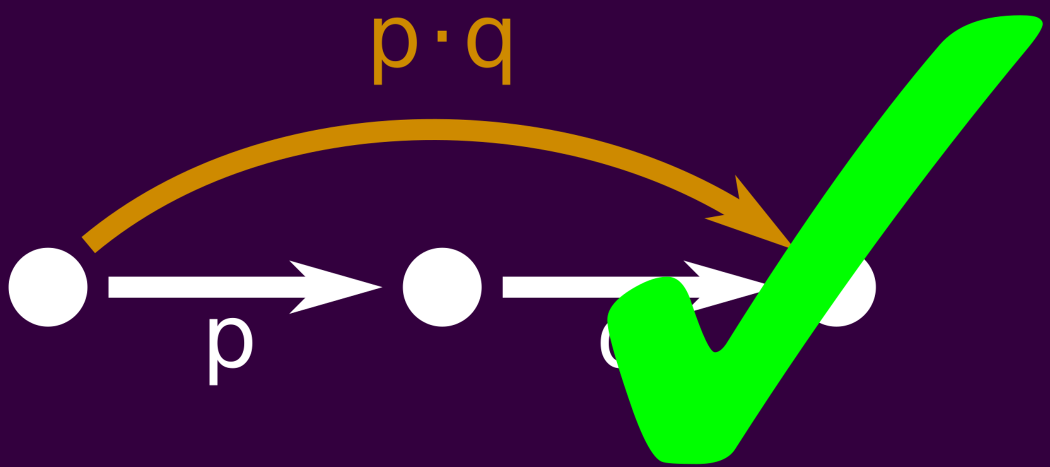

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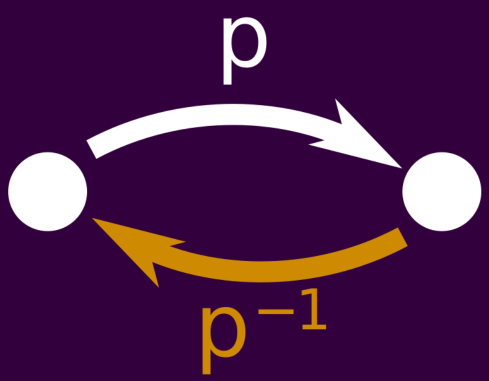

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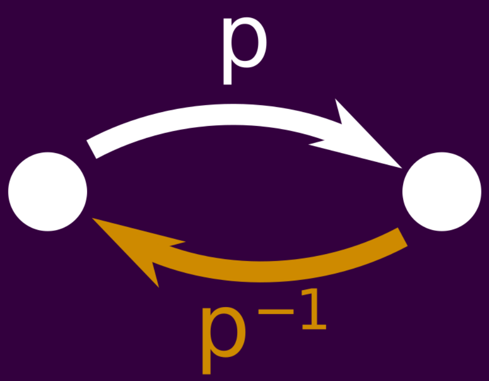

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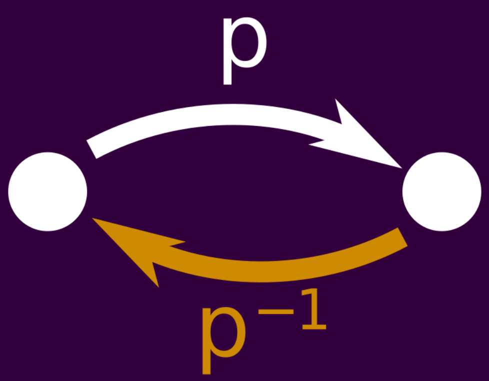

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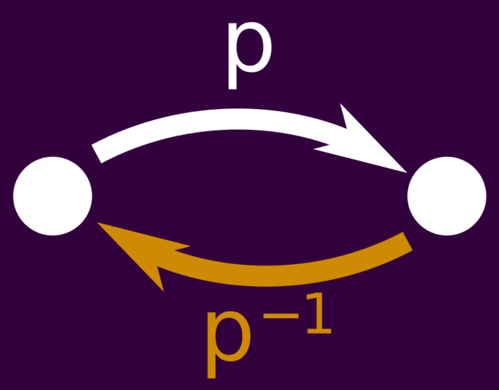

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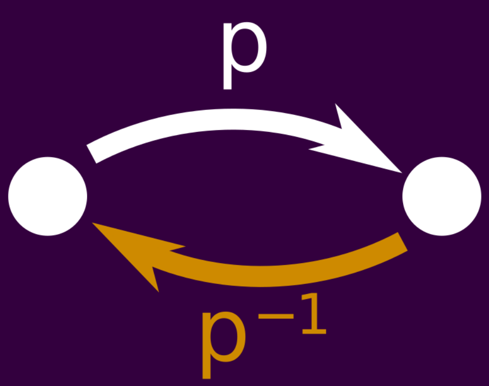

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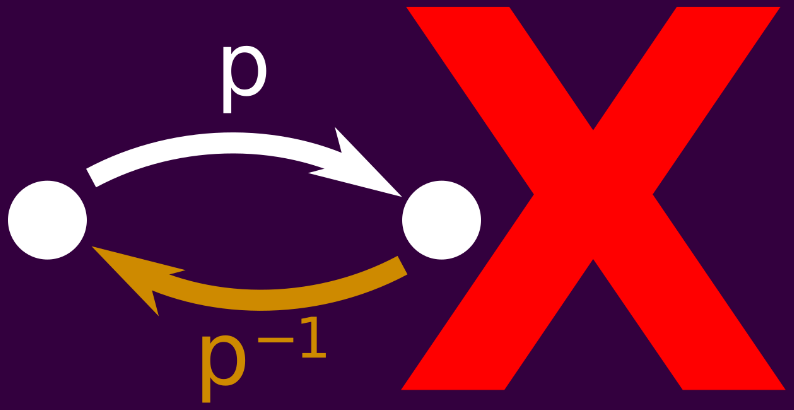

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Semantic Proof:

Symmetry *can't* be proved in general

Category theory is concerned with ***universal mapping properties***

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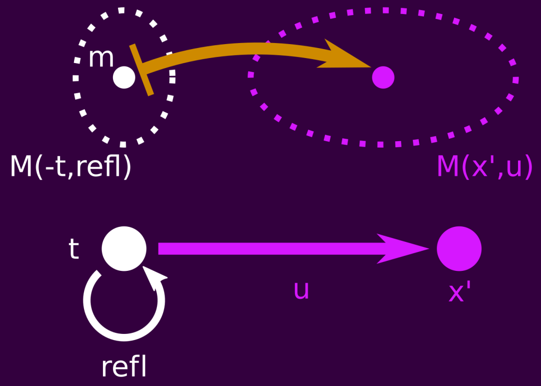
Type-theoretic constructs are introduced with
principles of induction



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Coproducts

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- $\iota_1 \colon \mathrm{Hom}(s, -Q)$ and
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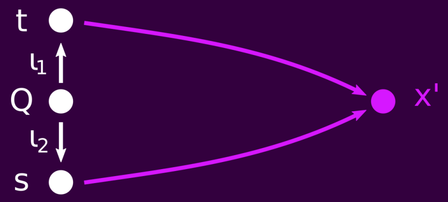
$$
\begin{tikzcd}
t \\
Q \arrow[u, "\iota_1"'] \arrow[d, "\iota_2"] \\
s
\end{tikzcd}
$$



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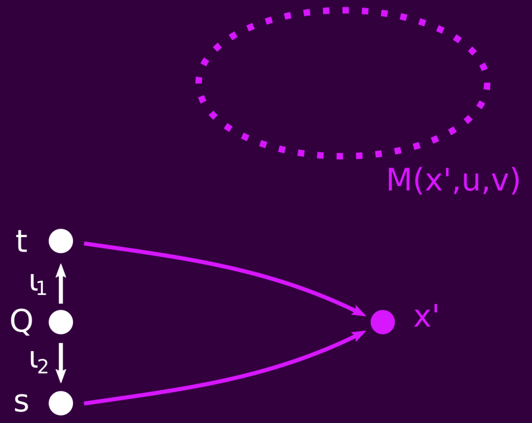
$$x' : A, u : \mathrm{Hom}(s, x'), v : \mathrm{Hom}(t, x') \vdash M \textbf{ type}$$



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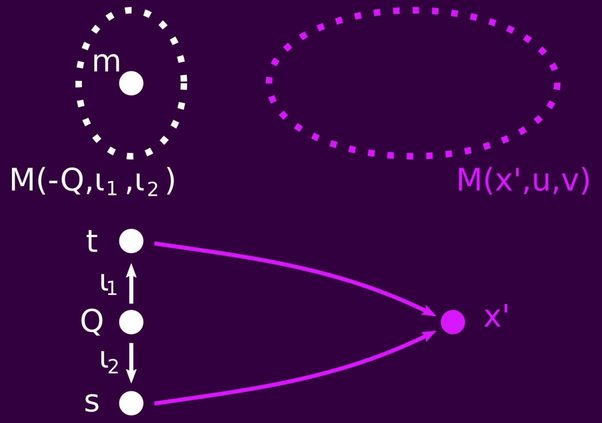

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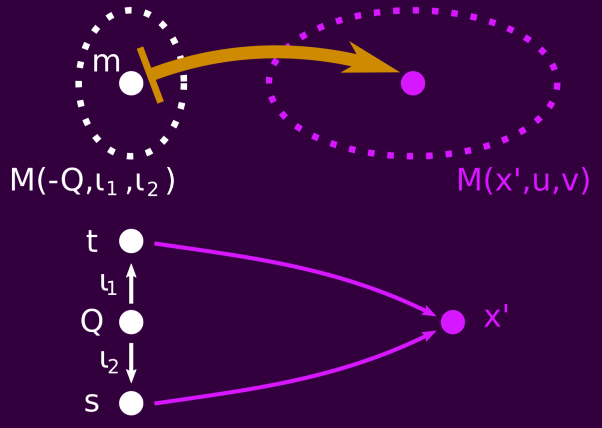

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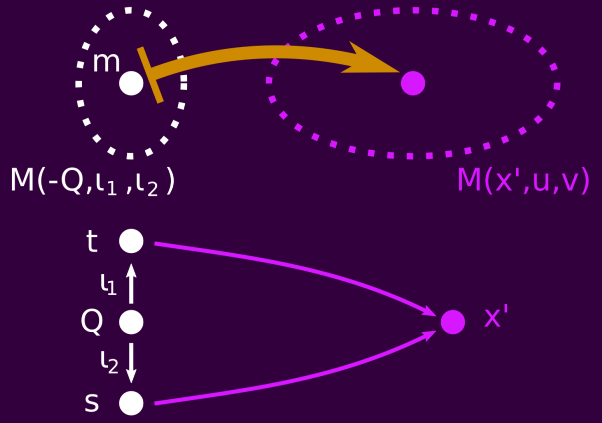

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Synthetic functors and natural transformations

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By Coslice Path Induction,

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$$\mathrm{elim}_M\ m\ (F(-t), \eta \,@\, (-t)) = m$$

Status

Apprehended

- Initial and Terminal Objects
- (Co)Products
- Pullbacks and Pushouts
- Left and Right Adjoints
- Applying a natural transformation
- Definition of Yoneda embedding
  - Exponentials
  - *All* limits of shape I

Still at large

- Lambda-abstraction rule for natural transformations
  - Proof by directed path induction that natural transformations are natural
  - Internal proof of Yoneda Lemma
  - (Co)limits in presheaf categories
- Monomorphisms/Epimorphisms (coinductive characterization?)

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[Neu25] Jacob Neumann.
A Generalized Algebraic Theory of Directed Equality.
PhD thesis, University of Nottingham, 2025.

[Nor19] Paige Randall North.
Towards a directed homotopy type theory.
Electronic Notes in Theoretical Computer Science, 347:223–239, 2019.

- Track polarities
- Limit coercions to closed terms
- Fail to prove symmetry
- Phrase universal mapping properties as principles of induction

Thank you!