A Fully Dependent Assembly Language

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TYPES 2025, University of Strathclyde, Glasgow

June 2025

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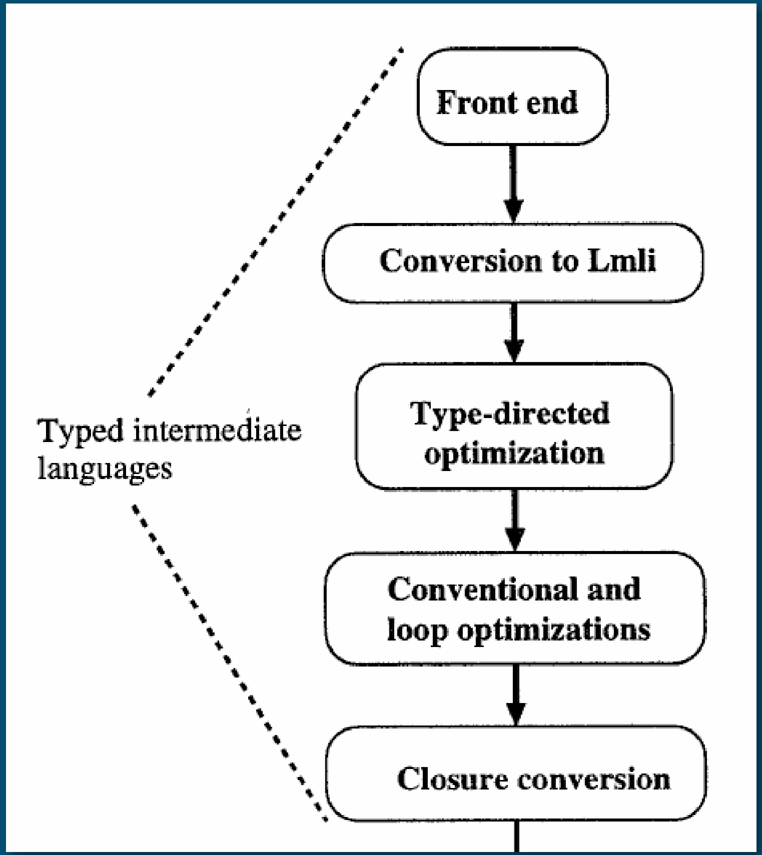

Tarditi et al. (1996): TIL: A type-directed optimizing compiler for ML

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- To verify executables through type-checking

```
l_fact:
    code[]{r1:⟨⟩,r2:int,r3:τ_k}.
      bnz r2,l_nonzero
      unpack [α,r3],r3
      ld r4,r3[0]
      ld r1,r3[1]
      mov r2,1
      jmp r4
```

Morrisett et al. (1999): From system F to typed assembly language

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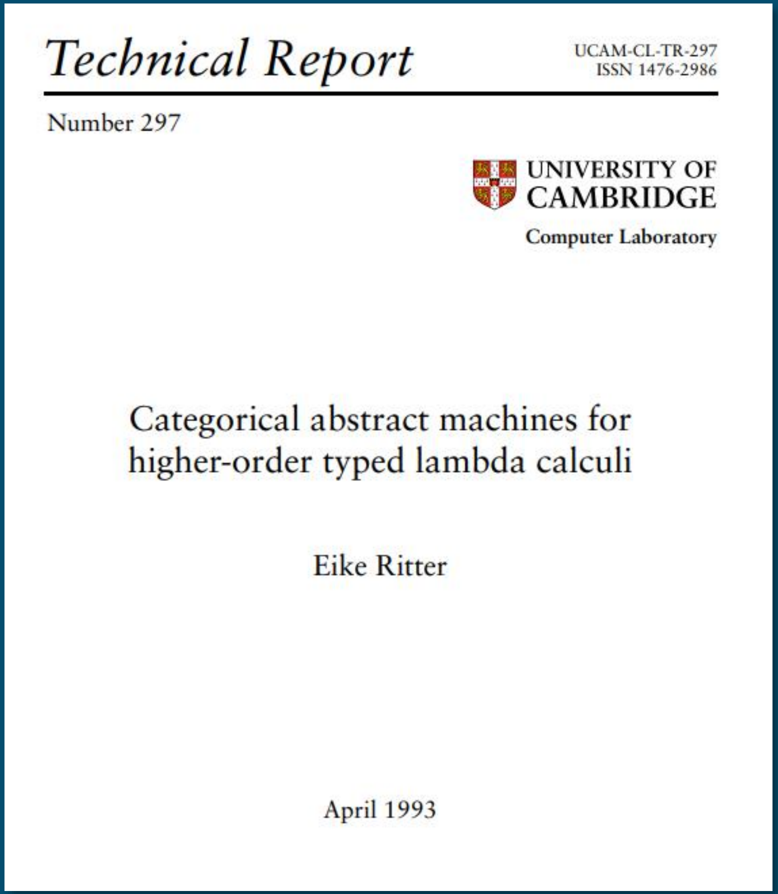
...and now is the time!



Technical Report

UCAM-CL-TR-297
ISSN 1476-2986

Number 297

UNIVERSITY OF CAMBRIDGE
Computer Laboratory

Categorical abstract machines for higher-order typed lambda calculi

Eike Ritter

April 1993

Ritter (1993): Categorical abstract syntax for higher-order typed lambda calculi

Dependent assembly how?

One should specify:

- how types depend on instructions
- the equational theory of instructions
- what is a dependent stack…

Dependent assembly how?

~~One should specify:~~

- ~~how types depend on instructions~~
- ~~the equational theory of instructions~~
- ~~what is a dependent stack...~~

One should separate high-level dependent types and low-level assembly code.

Dependent assembly: Syntax

Assembly: instruction set for a stack machine

$$I \;::=\; \texttt{LIT}\ c \mid \texttt{POP} \mid \texttt{VAR}\ x \mid \texttt{CLO}\ n\ lab \mid \texttt{APP} \mid I\ ;\ I' \mid \ldots$$

Term calculus: fully dependently typed calculus for specifying types of assembly code

$$e,\ A \;::=\; x \mid e\ e' \mid lab\ \{e_1, \ldots, e_n\} \mid \Pi x{:}A.B \mid U \mid \ldots$$

Term calculus: Defunctionalized CC

The term calculus is similar to calculus of constructions (CC) except:

- there is no lambda abstraction
- context contains a fixed set of function labels
- labels form closures with lists of terms

The defunctionalized CC is consistent and the function labels can be generated from a source program in CC.

$$\frac{\mathit{\Gamma} \vdash e_1, \ldots, e_n : \mathit{\Delta} \qquad lab(\mathit{\Delta},\, x{:}A \mapsto e : B) \in \mathit{\Gamma}}{\mathit{\Gamma} \vdash lab\{e_1, \ldots, e_n\} : (\mathit{\Pi} x{:}A.B)[e_1, \ldots, e_n \,/\, \mathit{\Delta}]}$$

Huang and Yallop (2023): Defunctionalization with dependent types

Assembly: SECD machine

Runtime values are closed values in the term calculus (i.e. closures, types, base values).

$$v \ ::= \ lab\,\{v_1,\ \ldots,\ v_n\} \mid A \mid \underline{b} \mid \ldots$$

A machine state $<I,\ Env,\ St,\ Fr>_P$ is made of:

- An instruction sequence (control) $\quad I$
- A runtime environment of values $\quad Env \ : \ List\ v$
- A stack of values $\quad St \ : \ List\ v$
- A stack of call frames (dump) $\quad Fr \ : \ List\,(I \times Env \times St)$
- A list of procedures $\quad P \ : \ Label \longrightarrow I$

Machine step: $\boxed{<I,\ Env,\ St,\ Fr>_P \ \longrightarrow\ <I',\ Env',\ St',\ Fr'>_P}$

Typing the assembly (judgement)

An abstract stack σ is a list of terms in the defunctionalized CC.

The typing judgement

$$\boxed{\Gamma \ \vdash I : \sigma \longrightarrow \sigma'}$$

says that instruction I transforms stack σ to stack σ', modelling the computation like an abstract interpreter.

Typing the assembly (basic operations)

$$\frac{x : A \in \Gamma}{\Gamma \vdash \texttt{VAR}\ x : \sigma \longrightarrow \sigma :: x}$$

$$< \texttt{VAR}\ x\ ;\ I,\ Env,\ St \qquad ,\ Fr>_P \longrightarrow$$
$$< I \qquad\qquad ,\ Env,\ St :: v,\ Fr>_P$$
$$\text{where } v = Env(x)$$

$$\frac{}{\Gamma \vdash \texttt{POP}\ : \sigma,\ t \longrightarrow \sigma}$$

$$< \texttt{POP}\ ;\ I,\quad Env,\ St :: v,\ Fr>_P \longrightarrow$$
$$< I \qquad ,\quad Env,\ St \qquad ,\ Fr>_P$$

Typing the assembly (closure)

`Clo` n lab forms a closure with the top n items on the stack.

$$\frac{\Gamma \vdash e_1, \ldots, e_n : \Delta \qquad lab(\Delta,\ x{:}A \mapsto e : B) \in \Gamma}{\Gamma \vdash \texttt{CLO}\ n\ lab : \sigma :: e_1 :: \ldots :: e_n \to \sigma :: lab\{e_1, \ldots, e_n\}}$$

$$\begin{aligned} &< \texttt{CLO}\ n\ lab\ ;\ I,\ Env,\ St :: v_1 :: \ldots :: v_n\ ,\ Fr >_P \longrightarrow \\ &< I\ ,\ Env,\ St :: lab\{v_1, \ldots, v_n\}\ ,\ Fr >_P \end{aligned}$$

Typing the assembly (application)

Application: loads instructions according to lab, fills in the environment, saves current (I, Env, St) on a new call frame.

$$\frac{\Gamma \vdash e : \Pi x{:}A.B \qquad \Gamma \vdash e' : A}{\Gamma \vdash \mathtt{APP} : \sigma :: e :: e' \;\rightarrow\; \sigma :: e\ e'}$$

$$\begin{array}{l} < \mathtt{APP}\ ;\ I,\ Env \qquad ,\ St :: lab\{Env'\} :: v,\ Fr >_P \longrightarrow \\ < P(lab)\ ,\ Env' :: v,\ [] \qquad ,\ Fr :: (I,\ Env,\ St)\ >_P \end{array}$$

Compilation

Now, we can define a simple compilation function that generates dependent assembly code from defunctionalized CC code:

$$
\begin{array}{lcl}
\mathsf{cp}\ \ x & = & \mathtt{VAR}\ x \\
\mathsf{cp}\ \ \mathit{\Pi x{:}A.B} & = & \mathtt{LIT}\ \mathit{\Pi x{:}A.B} \\
\mathsf{cp}\ \ U & = & \mathtt{LIT}\ U \\
\mathsf{cp}\ \ \mathit{lab}\ \{e_1,\ \ldots,\ e_n\} & = & \mathsf{cp}\ e_1\ ;\ \ldots\ ;\ \mathsf{cp}\ e_n\ ;\ \mathtt{CLO}\ n\ \mathit{lab} \\
\mathsf{cp}\ \ e\ e' & = & \mathsf{cp}\ e\ ;\ \mathsf{cp}\ e'\ ;\ \mathtt{APP}
\end{array}
$$

Correctness of compilation

Type preservation: $\Gamma \vdash e : A \implies \Gamma \vdash I : \sigma \to \sigma :: e$ *for all* σ.

Correctness (WIP): *For all base types* A, *if* $\cdot \vdash e : A$ *and* $e \rightsquigarrow^* v$, *then*

$$< \mathsf{cp}\ e, [], [], [] >_P \longrightarrow < [], [], [] :: v, [] >_P$$

Typing the machine states

Runtime values can be typed since they are closed values in the term calculus.

$\vdash v : A$

Other components of the machine state are also typable:

$\vdash Env : \Gamma$	env implements a context of type Γ
$\vdash_{Env} St : \sigma$	st implements σ w.r.t. well-formed Env

(judgements omitted for frames and procedures)

Above combine to a well-formedness judgement for machine states:

$\vdash < I, Env, St, Fr >_P$

Type safety

Progress:

$$If\ \vdash\ <I,\ Env,\ St,\ Fr>_P\ then\ <I,\ Env,\ St,\ Fr>_P\ \longrightarrow\ <I',\ Env',\ St',\ Fr'>_P$$

Preservation:

$$If\ \vdash\ <I,\ Env,\ St,\ Fr>_P\ and\ <I,\ Env,\ St,\ Fr>_P\ \longrightarrow\ <I',\ Env',\ St',\ Fr'>_P$$
$$then\ \vdash\ <I',\ Env',\ St',\ Fr'>_P$$

Future directions

- Termination
- Agda formalization of meta-theory
- Erasure of runtime types
- Certified optimization
- Datatypes and runtime representations
- Quantitative types for better erasure and linearity

Thank you!

…and questions?

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