A Timed Predicate Temporal Logic Sequent Calculus

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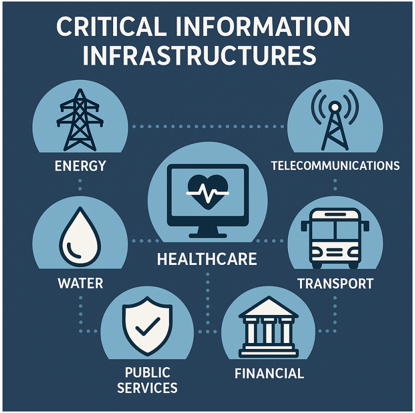
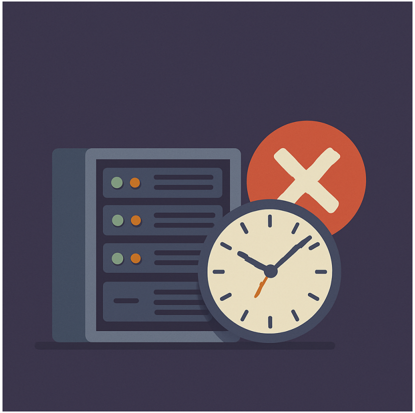
Motivation and Background



Time properties



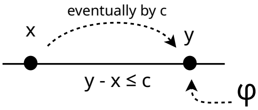
Temporal Logics

TPTL

TPTL is an extension of Linear Temporal Logic (LTL) & with explicit clock variables.

$$\varphi \quad ::= \quad p \mid \varphi \wedge \varphi \mid \neg\varphi \mid \varphi\, \mathbf{U}\, \varphi \mid \bigcirc\varphi \mid x \sim c \mid x \cdot \varphi$$

$$x \cdot \Diamond(x \leq c \wedge \varphi)$$



TPTL has been extended with past operators (TPTL + Past) **S** and **Y** .

Our Calculus

We present an extension of TPTL + Past

(1) Quantifiers: $\forall u : T.\varphi, \exists u : T.\varphi$
(2) General comparison operator: $t_1 \sim t_2$
(3) Discrete semantics based on timestamps t
(4) Formalized in Agda

Syntax

$$
\begin{array}{rcl}
\varphi & ::= & a \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi \mid \varphi \,\mathbf{U}\, \varphi \mid \bigcirc\varphi \mid \varphi \,\mathbf{S}\, \varphi \mid \mathbf{Y}\, \varphi \mid \\
 & & t \sim t \mid x \cdot \varphi \mid \forall u : T.\varphi \mid \exists u : T.\varphi \\
T & ::= & \mathbf{Agent} \mid \mathbf{Agents} \mid \mathbf{Data} \\
t & ::= & x \mid 0 \mid t \bullet t \mid \mathrm{s}(t) \\
\sim & ::= & < \mid = \\
a & ::= & p \mid \mathtt{send}(i, d, A) \mid \mathtt{recv}(i, d, j) \mid \mathtt{inter}(i, d)
\end{array}
$$

Semantics

Formulas are interpreted w.r.t.:

(1) t: a timestamp
(2) r: a function from timestamps to states
(3) π: an interpretation function
(4) v: a variable valuation

$$
\begin{array}{lcl}
\pi, r, t, v \models a & \iff & \pi(r(t))(a) \\
\pi, r, t, v \models \varphi \,\mathbf{U}\, \psi & \iff & \exists t' > t.(\pi, r, t', v \models \psi) \land \forall t'' \in [t, t').(\pi, r, t'', v \models \varphi) \\
\pi, r, t, v \models \bigcirc\varphi & \iff & \pi, r, t+1, v \models \varphi \\
\pi, r, t, v \models t_1 \sim t_2 & \iff & [\![t_1]\!]_v \sim [\![t_2]\!]_v \\
\pi, r, t, v \models x \cdot \varphi & \iff & \pi, r, t, v[x \mapsto t] \models \varphi \\
\pi, r, t, v \models \forall u : T.\varphi & \iff & \pi, r, t, v[u \mapsto z] \models \varphi \text{ for all } z \in [\![T]\!]
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\end{aligned}
$$

The Sequent Calculus

$\Gamma \vdash_{\mathsf{t}} \varphi$ states that φ holds at time t in the context Γ.

An hypothesis is of the form $(\psi)^r$, where r is a time annotation

$$\frac{\Gamma \vdash_{\mathsf{t}} \mathsf{t} < \mathsf{t}_1 \qquad \Gamma \vdash_{\mathsf{t}_1} \psi \qquad \Gamma, \mathsf{t} \leq x, x < \mathsf{t}_1 \vdash_x \varphi}{\Gamma \vdash_{\mathsf{t}} \varphi \, \mathbf{U} \, \psi} \ \mathbf{U}\,\mathsf{R}$$

$$\frac{\Gamma, \mathsf{t} < x, (\psi)^x, (\varphi)^{[\mathsf{t},x)} \vdash_{t_1} \gamma}{\Gamma, (\varphi \, \mathbf{U} \, \psi)^{\mathsf{t}} \vdash_{t_1} \gamma} \ \mathbf{U}\,\mathsf{L}$$

Expressiveness

Temporal operators such as "eventually" and "always" are defined as usual:

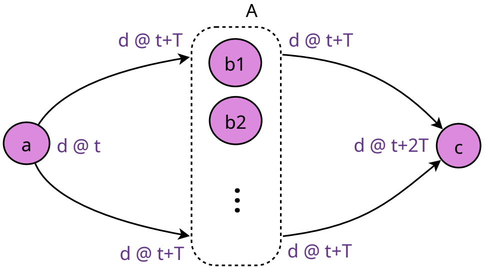
$$\Diamond\varphi := \texttt{true}\,\mathbf{U}\,\varphi \quad \text{and} \quad \Box\varphi := \neg\Diamond\neg\varphi$$

Time bounded version of the $\Diamond$ operator can now be defined:

$$\Diamond_t\varphi := x \cdot \Diamond(y \cdot y \leq x \bullet t \wedge \varphi)$$

Example

Assumptions:



$$\forall a : \textbf{Agent}.\forall d : \textbf{Data}.\forall A : \textbf{Agents}.\forall b : \textbf{Agent}.$$
$$\Box(\texttt{send}(a, d, A) \to \Diamond_T(b \in A \to \texttt{recv}(a, d, b)))$$

$$\forall a : \textbf{Agent}.\forall d : \textbf{Data}.\forall b : \textbf{Agent}.\forall c : \textbf{Agent}.$$
$$\Box(\texttt{recv}(a, d, b) \to \texttt{send}(b, d, \{c\}))$$

Conclusion:

$$\forall a : \textbf{Agent}.\forall d : \textbf{Data}.\forall b : \textbf{Agent}.\forall c : \textbf{Agent}.$$
$$\texttt{send}(a, d, \{b\}) \to \Diamond_{2T}(\texttt{recv}(b, d, c))$$

Takeaways

A timed predicate temporal logic

$$\Diamond_t \varphi := x \cdot \Diamond(y \cdot y \leq x \bullet t \land \varphi)$$

A labeled sequent calculus

$$\frac{\Gamma, \mathsf{t} < x, (\psi)^{x}, (\varphi)^{[\mathsf{t},x)} \vdash_{t_1} \gamma}{\Gamma, (\varphi \,\mathbf{U}\, \psi)^{\mathsf{t}} \vdash_{t_1} \gamma} \;\mathbf{U}\,\mathsf{L}$$

Discrete semantics

$$\pi, r, t, v \models x \cdot \varphi \iff \pi, r, t, v[x \mapsto t] \models \varphi$$

Application to distributed systems

$$\forall a : \mathbf{Agent}. \forall d : \mathbf{Data}. \forall b : \mathbf{Agent}. \forall c : \mathbf{Agent}.$$
$$\mathtt{send}(a, d, \{b\}) \to \Diamond_{2T}(\mathtt{recv}(b, d, c))$$

Future work: type system

Takeaways

A timed predicate temporal logic

$$\Diamond_t \varphi := x \cdot \Diamond(y \cdot y \leq x \bullet t \wedge \varphi)$$

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