An algebraic internal groupoidal model of Martin-Löf type theory

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Abstract

The groupoidal model of Martin-Löf type theory was introduced by Hofmann and Streicher in 1998, proving that the uniqueness of identity proofs property is false in this type theory and thus revealing higher dimensional structure [2]. In this model, dependent types are modelled by isofibrations between groupoids; these are functors with the *property* that paths can be lifted in a suitable way. Groupoids and isofibrations form a category with families, which gives a model of dependent type theory.

One issue with formalising models given by categories with families constructively is that substitution of terms is only modelled up to isomorphism rather than on the nose. Due to work by Gambino and Larrea [1], this problem can be ironed out by working with structure rather than mere properties.

In this talk, I will describe some recent work in which this algebraic approach is taken to construct a structural version of Hofmann and Streicher's groupoidal model. We work with cloven isofibrations; these are functors equipped with the *structure* of a path lifting. Moreover, we work in the setting of internal groupoid theory, without the reliance on set theoretic foundations. This recovers Hofmann and Streicher's approach when working internally to the category of sets and forgetting the algebraic structure. Our abstract setting allows for a relative consistency result: that models of Martin-Löf type theory can be constructed using Martin-Löf type theory.

References

[1] Nicola Gambino and Marco Federico Larrea. Models of Martin-Löf type theory from algebraic weak factorisation systems, *The Journal of Symbolic Logic*, 88(1):242–289, 2023.

[2] Martin Hofmann and Thomas Streicher. The groupoid interpretation of type theory. *Twenty- five years of constructive type theory (Venice, 1995)*, 36:83–111, 1998.